GOLDEN RATIO IN GEOMETRIC PROBLEMS

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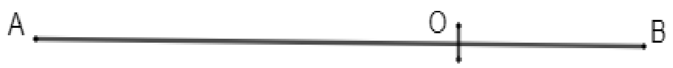
Abstract: Problems related to the ratio of triangles and sections that make up the golden ratio are given.

Key words: Elementary mathematics, planimetry, golden ratio, golden triangle.

Finding the ratio of sections is important in the introduction and study of geometrical problems. In the school geometry course, there are many concepts where the ratio of sections is determined, for example, similar triangles, Fales' theorem, determining the type of triangles, dividing the section in a given ratio, and so on. In triangles, the point where the medians intersect is in the ratio 2:1 from the tip of the median, the bisector of a triangle intersects the side it falls on, and the sides it adjoins in the ratio. We are below $\frac{\sqrt{5}+1}{2}$ Let's consider the geometrical problems in which the sections defined in the ratio form the golden ratio.

Let us be given the following section AB. We need to divide this section into two parts. Let one be small and one big. Let us denote the point of division as O:



Here, let's choose the point O so that the ratio of the length of the large section AO to the length of the small section OB is equal to the ratio of the length of the initial section AB to the length of the large section AO, that is:

$$AO:OB = AB:AO$$

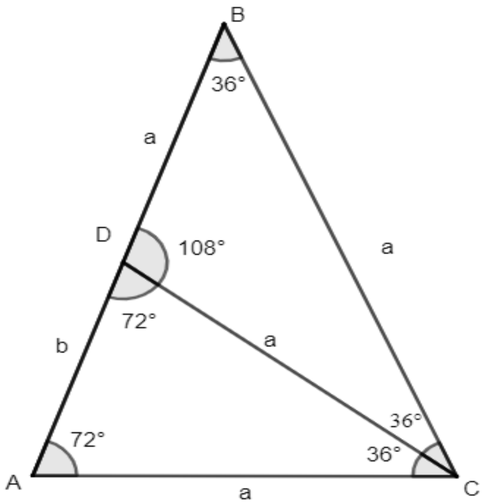
The number formed in this ratio is defined as (pronounced 'phi'-“fi”)."φ"

$$\varphi = AO:OB = AB:AO \qquad (1)$$

$AB = a, AO = x$if we take it, it will be. From equality (1) we arrive at the quadratic equation. Solving this, we find that The value of this number is equal to . (This number is an infinite non-recurring decimal like the number we all know. Here, 13 numbers are given after the comma). Point O is called golden center or golden point for section AB.$OB = a - xx^2 + ax - a^2 = 0\varphi = \frac{\sqrt{5}+1}{2}\varphi = 1.618033988749895 \ldots \varphi\pi\varphi$

The concept of the golden triangle and its application

A triangle with angles is called a golden triangle. It has the following remarkable property: Suppose triangle ABC is a golden triangle (Figure 1). Here$72^\circ, 36^\circ, 72^\circ$



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$\angle A = 72^\circ, \angle B = 36^\circ, \angle C = 72^\circ$. Let's make a cut CD that bisects the angle C. The resulting COD will also be the angles of the triangle. Here, point D is divided by the golden ratio.$72^\circ, 72^\circ, 36^\circ$

$\Delta ABC \sim \Delta ADC$these two triangles are similar.

Now using the sign of equality,

$\frac{a}{b} = \frac{a+b}{a}$we will have equality. From this$\frac{a}{b} = 1 + \frac{b}{a}$

$\frac{a}{b} = t, \frac{b}{a} = \frac{1}{t}$if we introduce the notation, we get the equation.$t = 1 + \frac{1}{t}$

$$t^2 - t - 1 = 0$$

$t_{1.2} = \frac{1 \pm \sqrt{5}}{2}$, if we take a positive value from it.

$t = \frac{1+\sqrt{5}}{2}, \ \frac{a}{b} = \frac{1+\sqrt{5}}{2};$

ΔADCif we apply the theorem of sines in a triangle:

$$\frac{b}{\sin 36^\circ} = \frac{a}{\sin 72^\circ}$$

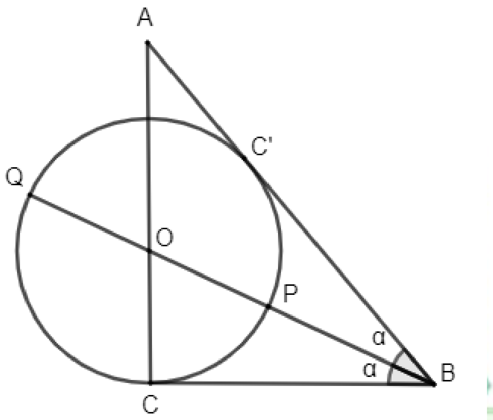
$$\frac{\sin 72^\circ}{\sin 36^\circ} = \frac{a}{b} \Rightarrow 2\cos 36^\circ = \frac{a}{b}, \quad 2\cos 36^\circ = \frac{1+\sqrt{5}}{2}$$

$\cos 36^{\circ} = \frac{1+\sqrt{5}}{4}$, , from Eq$\cos 36^{\circ} = \cos(2 \cdot 18)^{\circ} = 1 - 2 \cdot \sin^2 18°$

$$\frac{1+\sqrt{5}}{2} = 1 - 2 \cdot \sin^2 18°, \qquad \sin^2 18° = \frac{1-\sqrt{5}}{4}, \qquad \cos^2 18° = \frac{3+\sqrt{5}}{4};$$

$\cos 18^{\circ} = \frac{\sqrt{3+\sqrt{5}}}{2}$, $\cos 18^{\circ} = \cos(2 \cdot 9)^{\circ} = 1 - 2 \cdot \sin^2 9°$, $\frac{\sqrt{3+\sqrt{5}}}{2} = 1 - 2 \cdot \sin^2 9°$,

$\sin^2 9° = \frac{2-\sqrt{3+\sqrt{5}}}{4}$, $\cos^2 9° = \frac{2+\sqrt{3+\sqrt{5}}}{4}$, $\cos 9^{\circ} = \frac{\sqrt{2+\sqrt{3+\sqrt{5}}}}{2}$, $\sin 9^{\circ} = \frac{\sqrt{2-\sqrt{3+\sqrt{5}}}}{2}$It follows that using the golden ratio and the golden triangle, it is possible to find the trigonometric values of angles that are multiples of 9.



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Issue 1.

ΔABCgiven a triangle (Fig. 2), its sides are corresponding

. Angle bisector B intersects side AC at point O. A circle with center O and radius OC is drawn, the bisector BO intersects the circle at points P and Q. show that the ratios are equal and the golden ratio.$AB = 5, AC = 4\ va\ \ BC = 3 \frac{PQ}{BP} = \frac{BQ}{PQ}$

Solution: In this triangle, the Pythagorean theorem is valid, so our triangle is a right triangle. In our circle, lar is the radius of the circle, dir.$AB^2 = AC^2 + BC^2 \Delta ABCQO, OP, OCQO = OP = OC = r$

$$OB = l_{AC} = \frac{\sqrt{AB \cdot BC(AB+BC+AC)(AB+BC-AC)}}{AB+BC} = \frac{\sqrt{3 \cdot 5(3+5+4)(3+5-4)}}{3+5} = \frac{3\sqrt{5}}{2}$$

$\Delta OCB$is also a right triangle. From this:

$$OB^2 = OC^2 + BC^2, \quad OC^2 = \left(\frac{3\sqrt{5}}{2}\right)^2 - (3)^2 = \frac{9}{4}, \qquad OC = \frac{3}{2} = r$$

$$OC + OP = OB \ \ bo'lsa \qquad r + x = l_{AC},$$

$$x = l_{AC} - r = \frac{3\sqrt{5}}{2} - \frac{3}{2} = \frac{3\sqrt{5}-3}{2}, \quad BQ = 2r + x = 2\frac{3}{2} + \frac{3\sqrt{5}-3}{2} = \frac{3}{2}(\sqrt{5}+1)$$

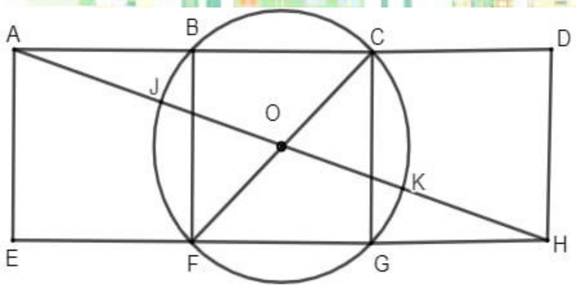
$$\frac{QP}{PB} = \frac{2r}{x} = \frac{2 \cdot \frac{3}{2}}{\frac{3\sqrt{5}-3}{2}} = \frac{\sqrt{5}+1}{2} = 1.618033 \ldots$$

$$\frac{BQ}{PQ} = \frac{\frac{3}{2}(\sqrt{5}+1)}{2\frac{3}{2}} = \frac{\sqrt{5}+1}{2} = 1.618033 \ldots$$

So $\frac{QP}{PB} = \frac{BQ}{PQ} = \frac{\sqrt{5}+1}{2} = 1.618033 \ldots$

Issue 2: Show that the ratio of the diameter of the outer circle JK to AJ or KH is the golden ratio in the square in which the diagonal of the rectangle ADHE is passed from the point A to the point H and the side is equal to a. picture).

Solution: $AB = BC = CD = DH = HG = GF = FE = AE = a;\ \ AJ = KH;$



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$$AH^2 = AE^2 + EH^2$$

$$AH^2 = a^2 + (3a)^2$$

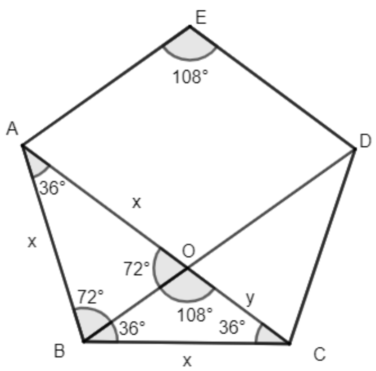
$$AH = a\sqrt{10}$$

$$FC = a\sqrt{2} = JK$$

$$AJ = \frac{AH - JK}{2} = \frac{a\sqrt{10} - a\sqrt{2}}{2}$$

$$\frac{JK}{AJ} = \frac{a\sqrt{2}}{\frac{a\sqrt{2}(\sqrt{5}-1)}{2}} = \frac{2}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{2} = \varphi = 1.618033 \ldots.$$

Issue 3. *If the intersecting diagonals of an arbitrary arrangement of pentagons intersect in the golden ratio from the point of intersection* $\frac{x}{y}$=?(Figure 4)



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The solution

$\Delta BAC \sim \Delta OBC$ from similarity $\frac{x+y}{x} = \frac{x}{y} \Rightarrow \frac{x}{y} = 1 + \frac{y}{x},\quad \frac{x}{y} = t,\quad \frac{y}{x} = \frac{1}{t},$

$t = 1 + \frac{1}{t}$ we get the equation , if we take a positive value from it. $t^2 - t - 1 = 0$, $t_{1.2} = \frac{1 \pm \sqrt{5}}{2} t = \frac{1+\sqrt{5}}{2},\quad \frac{x}{y} = \frac{1+\sqrt{5}}{2};\quad \frac{x}{y} = 1.618033 \ldots$

This is evidence of the golden ratio. The golden ratio is so important not only because it has amazing properties in geometry, but also because of its many occurrences in nature. In addition, the golden ratio is important in painting, music, architecture and many other fields.

In conclusion, it can be said that this universe around us is built on the basis of certain precise measurements, so that every object in it works as a perfect system, and such a system never appeared in a random state. Each particle in the universe is carrying out the order assigned to it without deviation, based on certain rules, and behind this is the great creator god who created everything in measure.

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